**Air Force Institute of Technology**

**Graduate School of Engineering and Management**

**Department of Electrical and Computer Engineering**

**CSCE 532 Automata and Formal Languages**

**Winter 2019**

# Day 1 – Introduction

§0.2 Mathematical Notations and Terminology

1. List as many differences as you can between Sipser’s and Rosen’s notation and terminology.

§0.3 Definitions, Theorems, and Proofs

1. (Sipser Problem 0.10) Find the error in the following “proof” that .

Consider the equation . Multiply both sides by a to obtain . Subtract from both sides to get . Now factor each side, , and divide each side by to get . Finally, let and equal , which shows that .

The step involving division by requires that , which is violated by letting and equal .

1. (Sipser Problem 0.11) Let be the sum of the first natural numbers and let be the sum of the first cubes. Prove the following equalities by induction on , to arrive at the curious conclusion that for every .
2. .

Proof: By induction on .

Basis step (): .

Inductive hypothesis: .

Induction step: Show .

1. .

Proof: By induction on .

Basis step (): .

Inductive hypothesis: .

Induction step: Show .

1. (Sipser Problem 0.12) Find the error in the following proof that all horses are the same color.

CLAIM: In any set of horses, all horses are the same color.

PROOF: By induction on .

Basis: For . In any set containing just one horse, all horses clearly are the same color.

Induction step: For , assume that the claim is true for and prove that it is true for. Take any set of horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set with just horses. By the induction hypothesis, all the horses in are the same color. Now replace the removed horse and remove a different one to obtain the set . By the same argument, all the horses in are the same color. Therefore, all the horses in must be the same color, and the proof is complete.

The basis step (for ) is correct. However, the induction step is not valid for the case , i.e. the truth of the claim for does not imply its truth for . Specifically, consider . Removing yields , while instead removing yields . We have , and indeed all the horses in are the same color (Dark bay) and all the horses in are the same color (Chestnut). However, it is not the case that all the horses in are the same color.